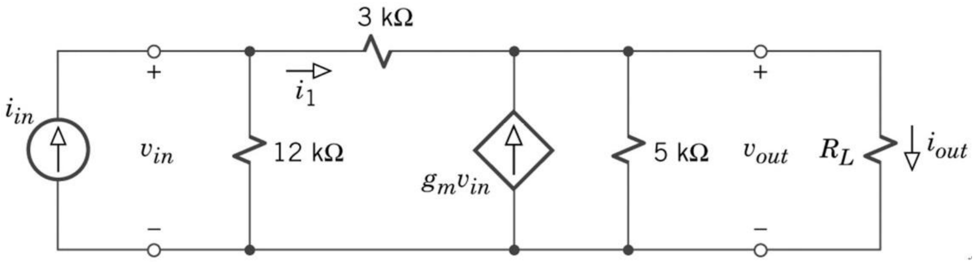
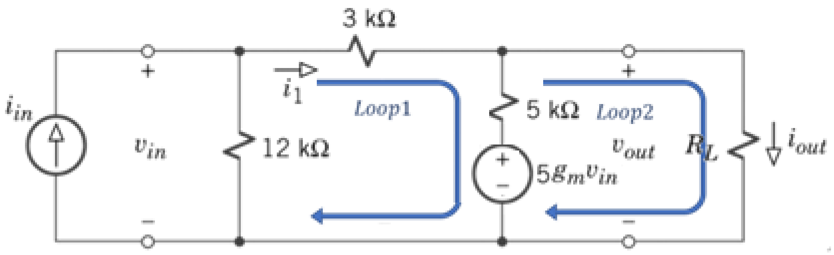
2025 Spring - Circuits - Final

1. (15 pts) For the circuit below, g_m = 1 mS and R_L = 1 kΩ. Please obtain:
   (a) (10 pts) the matrix mesh equation for i_1 and i_{out}.
   (b) (5 pts) current gain i_{out} / i_{in}.



Solution



(a)

Loop1 : $-12(i_1 - i_n) - 3i_1 - 5g_m v_{in} + 5(i_{out} - i_{in}) = 0$ (2pts)

Loop2: $-5(i_{out} - i_1) + 5g_m v_{in} - i_{out} = 0$ (2pts)

$v_{in} = 12(i_{in} - i_1)$ (2pts)

$g_m v_{in}$代入$Loop1, Loop2$

Loop1 $\Rightarrow 40i_1 + 5i_{out} = 48i_{in}$ (1pts)

Loop2 $\Rightarrow 55i_1 + 6i_{out} = 60i_{in}$ (1pts)

A: $\begin{bmatrix} 40 & 5 \\ 55 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_{out} \end{bmatrix} = \begin{bmatrix} 48 \\ 60 \end{bmatrix} i_{in}$ (2pts)

(b)

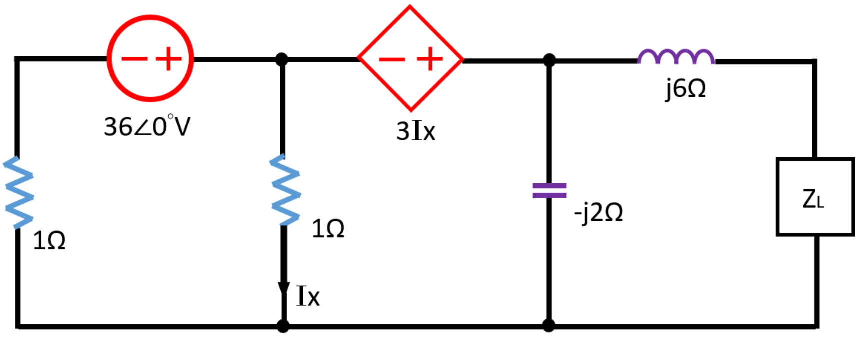
$$\begin{bmatrix} 40 & 5 \\ 55 & 6 \end{bmatrix} \begin{bmatrix} \frac{i_1}{i_{in}} \\ \frac{i_{out}}{i_{in}} \end{bmatrix} = \begin{bmatrix} 48 \\ 60 \end{bmatrix}$$ (2pts)

A: $\frac{i_{out}}{i_{in}} = \frac{\Delta_2}{\Delta} = \frac{det\begin{bmatrix} 40 & 48 \\ 55 & 60 \end{bmatrix}}{det\begin{bmatrix} 40 & 5 \\ 55 & 6 \end{bmatrix}} = \frac{48}{7} = 6.86$ (3pts)

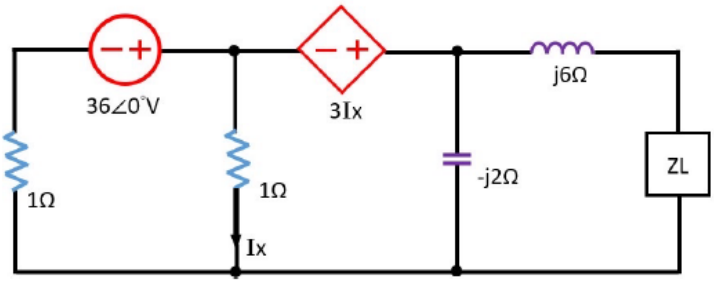
2. (7 pts) For the circuit below with load $\mathbf{Z_L}$, find the following:

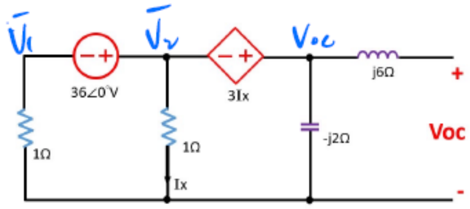
   (a) (4 pts) the load impedance $\mathbf{Z_L}$ as a complex number for maximum average power transfer to the load

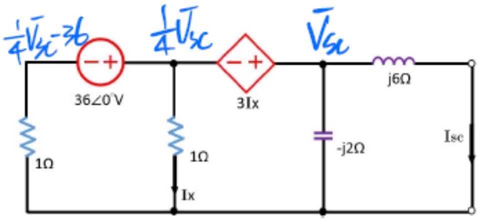
   (b) (3 pts) the value of the maximum average power absorbed by the load



Solution







$$\frac{\bar{V}_{oc} - 3\bar{I}_x}{1} = \bar{I}_x \Rightarrow \bar{I}_x = \frac{1}{4}\bar{V}_{oc}$$

$$\Rightarrow \bar{V}_2 = \bar{V}_{oc} - 3\left(\frac{1}{4}\bar{V}_{oc}\right) = \frac{1}{4}\bar{V}_{oc}$$

$$\Rightarrow \bar{V}_1 = \frac{1}{4}\bar{V}_{oc} - 36\angle 0^\circ \text{ V}$$

$$\frac{\frac{1}{4}\bar{V}_{oc} - 36\angle 0^\circ}{1} + \frac{\frac{1}{4}\bar{V}_{oc}}{1} + \frac{\bar{V}_{oc}}{-j2} = 0$$

$$\Rightarrow \bar{V}_{oc} = 36\sqrt{2}\angle -45^\circ$$

$$\frac{\bar{V}_{sc} - 3\bar{I}_x}{1} = \bar{I}_x \Rightarrow \bar{I}_x = \frac{1}{4}\bar{V}_{sc}$$

$$\frac{\frac{1}{4}\bar{V}_{sc} - 36\angle 0^\circ}{1} + \frac{\frac{1}{4}\bar{V}_{sc}}{1} + \frac{\bar{V}_{sc}}{-j2} + \frac{\bar{V}_{sc}}{j6} = 0$$

$$\Rightarrow \bar{V}_{sc} = 59.9\angle -33.69^\circ \text{ V}$$

$$\Rightarrow \bar{I}_{sc} = 9.9846\angle -123.69^\circ \text{ A}$$

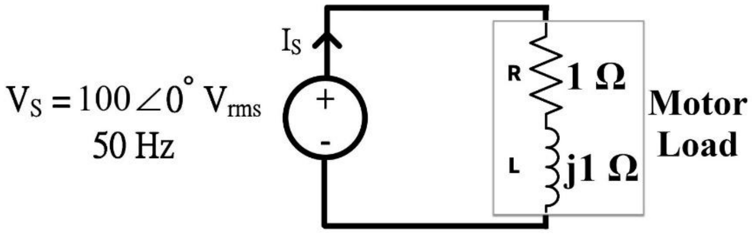
$$\Rightarrow Z_{th} = \frac{\bar{V}_{oc}}{\bar{I}_{sc}} = 1 + j5$$

$$\Rightarrow Z_{th}^* = 1 - j5$$

$$Z_L = 1 - j5\ \Omega = \sqrt{26}\angle -78.69\ \Omega$$

$$P_{max} = \frac{V_{oc}^2}{4} = 648\text{ W}$$

3. (8 pts) The circuit below operates at 50 Hz and is in steady state. The phasor value is the RMS (not magnitude). The load is a motor with the shown equivalent impedance. Find the following:
   (a) (2 pts) real power of the load, **P**
   (b) (2 pts) reactive power of the load, **Q**
   (c) (2 pts) power factor of the load, **PF**
   (d) (2pts) value of the capacitor **C** that, when put in parallel with the load, makes the supply power factor become unity (PF=1.0)



Solution

(a) $Z = 1 + j1 = 1.41 \angle 45° \ \Omega$

$I = V/Z = 100 \angle 0° / 1.41 \angle 45° = 70.7 \angle -45°$ A

$P = V_m I_m \cos(\phi) = 100 * 70.7 \cos(45°) = 4{,}999.2$ W (2 pts)

(b) $Q = V_m I_m \sin(\phi) = 100 * 70.7 \sin(45°) = 4{,}999.2$ VAR (2 pts)

(c) $PF = \cos(\phi) = \cos(45°) = 0.707$ (2 pts)

(d) $Z_C = -j\ 1/(2\pi f C) = 1/(100\pi C) \angle -90° \ \Omega = 0.00318/C \angle -90° \ \Omega$

$I_C = V/Z_C = 100 \angle 0° / 0.00318/C \angle -90° = 31{,}416\ C \angle 90°$ A

$Q_C = V_m I_m \sin(\phi) = 100 * 31{,}416\ C \sin(-90°) = -3{,}141{,}600\ C$

$C = Q_C / -3{,}141{,}600 = -4{,}999.2 / -3{,}141{,}600 = 0.001591$ F $= 1.591$ mF (2 pts)

4.

(a) (8 pts) You are asked to design a second-order bandpass filter with peak gain of one and passing frequency in the range of 2 kHz ± 100 Hz *(i.e.,* f_l = 1.9 kHz and f_u = 2.1 kHz). Write down the transfer function $H_{bp}(s)$ of this filter and the corresponding pole locations. Sketch the frequency response curves (amplitude and phase) for $H_{bp}(s)$ and specify the amplitude $a(f)$ and phase $\theta(f)$ at: $f = 0+, f_l, f_0, f_u, \infty$.

(b) (4 pts) Design a first-order high pass filter with peak gain of one and cutoff frequency f_{co} = 1.9 kHz, and a first-order low pass filter with peak gain of one and cutoff frequency f_{co} = 2.1 kHz. Write down the corresponding transfer functions $H_{hp}(s)$ and $H_{lp}(s)$.

(c) (3 pts) One engineer tries to combine the high pass and low pass filters you designed in (b) to implement the second-order bandpass filter you need in (a). Would it work? Why or why not?

Solution

4(a)

For $f_l = 1.9k$ Hz, $f_u = 2.1k$ Hz, $f_0 = 2k$ Hz

The second order transfer function is given by:

$$H_{bp}(s) = \frac{Ks}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Where $\omega_0 = 2\pi f_0 = 12566.4\ (rad/s)$,

Q is the quality factor, defined as $Q = \frac{\omega_0}{B} = \frac{\omega_0}{\omega_u - \omega_l} = \frac{2000*2\pi}{(2100-1900)*2\,pi} = 10$

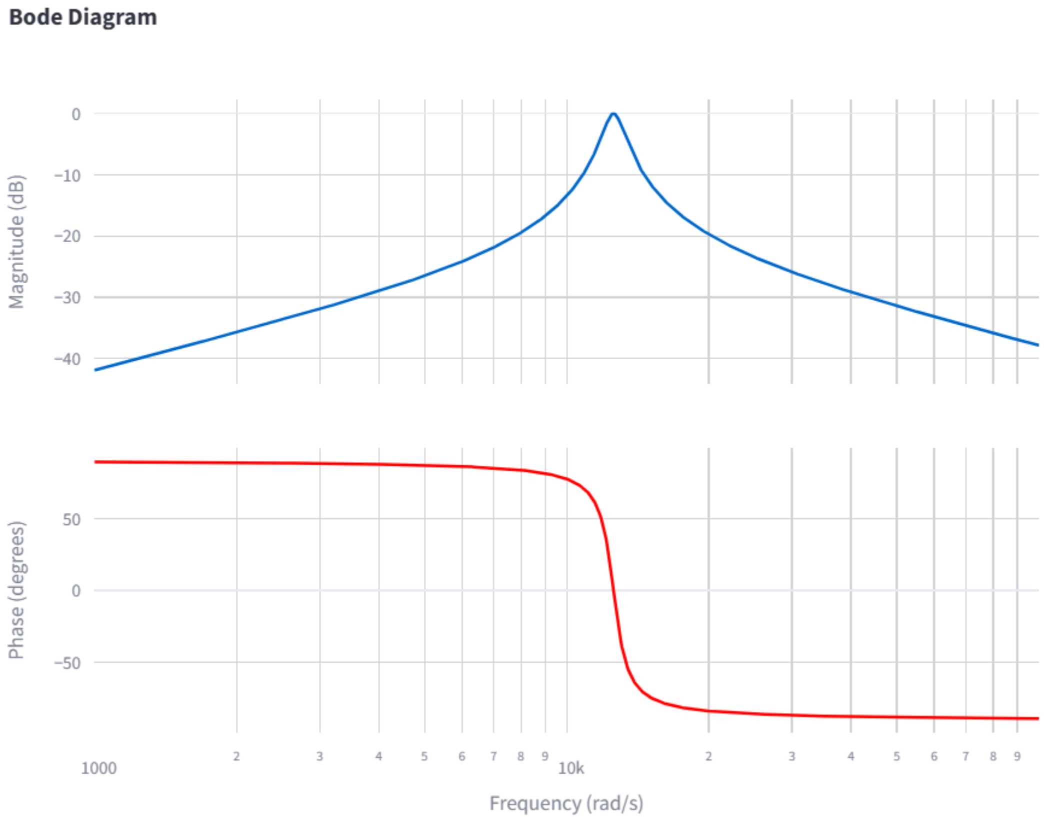
For peak gain = 1, at $f = f_0$, $K = \frac{\omega_0}{Q} = 1256.64$

So the transfer function becomes:

$$H_{bp}(s) = \frac{1256.64s}{s^2 + 1256.64s + 12566.4^2}$$

Pole locations :

$$s^2 + 1256.64s + 12566.4^2 = 0$$
$$\Rightarrow s = -628.32 \pm j12550.7$$



f	0^+	f_l	f_0	f_u	∞
$a(f)$	0	-3dB	1	-3dB	0
$\theta(f)$	90°	45°	0°	$-45°$	$-90°$

4(b)

First order high-pass filter with unity gain is:

$$H_{hp}(s) = \frac{s}{s + \omega_l} = \frac{s}{s + 2\pi \cdot 1.9k} = \frac{s}{s + 11938}$$

First order low-pass filter with unity gain is:

$$H_{lp}(s) = \frac{\omega_u}{s + \omega_u} = \frac{2\pi \cdot 2.1k}{s + 2\pi \cdot 2.1k} = \frac{13195}{s + 13195}$$

4(c) It doesn't work.

In part (a), we carefully scaled the filter to get unity gain at f_0. In the cascaded filter as (b), the gain at f_0 is **less than 1**, and varies depending on the specific cutoff frequencies of the HPF and LPF.

Hence, this cascade filter **does not** match the desired filter in part (a).

評分標準:
4(a). $H_{bp}(s)$ 寫對得 3 分，求出Q, K各 1 分。
Poles 寫對 1 分。
Bode plot 2 分 (Amplitude, Phase 各 1 分), 表格 2 分(Amplitude, Phase 各 1 分)。
共 8 分。

4(b). $H_{hp}(s)$ 寫對得 2 分，$H_{lp}(s)$ 寫對得 2 分，共 4 分。

4(c). 寫對理由得 3 分，只寫 No

5. (9 pts) $F(s)$ is the Laplace transform of $f(t)$. Given $F(s)=\frac{-3s^3+2}{(s^2+1)(s^2+6s+5)}$ and $f(0^-)=-3$. Evaluate (a) $f(0^+)$; (b) $f'(0^+)$; (c) $f(\infty)$. Be sure to show the detailed steps taken to obtain the answer.

Solution

Method 1

1. Given the Laplace transform

$$F(s)=\frac{-3s^3+2}{(s^2+1)(s^2+6s+5)}=\frac{-3s^3+2}{(s^2+1)(s+1)(s+5)}$$

$$F(s)=\frac{(\mathrm{A}\,s+\mathrm{B})}{(s^2+1)}+\frac{\mathrm{C}}{(s+1)}+\frac{\mathrm{D}}{(s+5)}$$

get A = 0, B = 1/2, C = 5⁄8, D = −29⁄8. (1)

Inverse Laplace: $f(t)=\frac{1}{2}\sin t+\frac{5}{8}e^{-t}-\frac{29}{8}e^{-5t}\quad t\geq 0.$ (2)

F $(0^+) = -3$ (2)

f' $(0^+) = 18.$ (2)

The undamped sin t‘ term prevents convergence ⇒ f(∞) is undefined. (2)

Method 2: Initial and Final Value Theorems

Given:

1. **Initial Value Theorem**

   $f(0+) = \lim_{s \to \infty} sF(s)$

   $f(0+) = -3,$ (3)

2. **Initial Derivative**

   If you also need the slope at t=0+t=0+, use the property

   $f'(0+) = \lim_{s \to \infty} [s^2 F(s) - s\, f(0+)]$

   Substituting f(0+)=−3 and F(s) get

   $f'(0+) = 18,$ (3)

3. **Final Value Theorem**

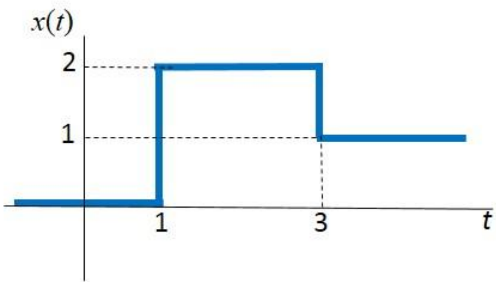
   $f(\infty) = \lim_{s \to 0} sF(s)$

   F(s) has poles on the imaginary axis (specifically at $s = \pm j\omega$, the roots of $s^2+1=0$), so this limit does **not** exist.

   $\lim_{s \to 0} sF(s)$ diverges because of poles at $s = \pm j\omega$, $f(\infty)$ is undefined. (3)

6. (12 pts) An s-domain circuit diagram yields the transfer function H(s) = Y(s)/X(s). Find the zero-state response $y(t)$ when input $x(t)$ is as shown in the figure below.

$\mathrm{H(s)} = \frac{10s^2+300s}{(s+10)(s^2-100)}$



Solution

Transfer function: $H(s) = \frac{10s^3+300\mathrm{s}}{(s^2-100)(\mathrm{s}+10)}$

Input waveform: x(t) = 2 u(t − 1) − u(t − 3). (2)

Laplace transform:

$X(s) = \frac{2e^{-\mathrm{s}}-e^{-3\mathrm{s}}}{\mathrm{s}}$ (2)

$$H(s) = \frac{10\mathrm{s}(\mathrm{s}+30)}{(\mathrm{s}+10)^2(\mathrm{s}\ -\ 10)}$$

Define $G(s) = \frac{10(\mathrm{s}+30)}{(\mathrm{s}+10)^2(\mathrm{s}-10)}$ Then Y(s) =X(s)H(S) = $\frac{2e^{-\mathrm{s}}-e^{-3\mathrm{s}}}{\mathrm{s}}$ H(s) =

$2e^{-\mathrm{s}} - e^{-3\mathrm{s}}$ G(s)

$G(s) = \frac{\mathrm{A}}{(\mathrm{s}-10)} + \frac{\mathrm{B}}{(\mathrm{s}+10)} + \frac{\mathrm{C}}{(\mathrm{s}+10)^2}$ ⇒ A = 1, B = −1, C = −10.

g(t) =$e^{10\mathrm{t}}$-$e^{-10\mathrm{t}}$-10 t$e^{-10\mathrm{t}}$, t ≥ 0. (2)

y(t) = 2 g(t − 1) u(t − 1) − g(t − 3) u(t − 3). (3)

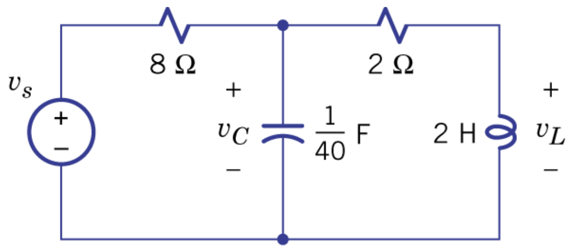
y(t) = 2[$e^{10(\mathrm{t}-1)}$-$e^{-10(\mathrm{t}-1)}$-10 $(\mathrm{t}-1)e^{-10(\mathrm{t}-1)}$]u(t − 1) − [$e^{10(\mathrm{t}-3)}$-$e^{-10(\mathrm{t}-3)}$-10 $(\mathrm{t}-3)e^{-10(\mathrm{t}-3)}$] u(t − 3). (3)

7. (34 pts) Consider the dynamic circuit below. The time function of the voltage source is $v_s(t) = \begin{cases} 20\ V & t < 0 \\ 8e^{-t}\ V & t > 0 \end{cases}$. Use the <u>Laplace transform method</u> to analyze the circuit.

(a) (14 pts) Construct and plot the s-domain circuit diagram for t ≥ 0.

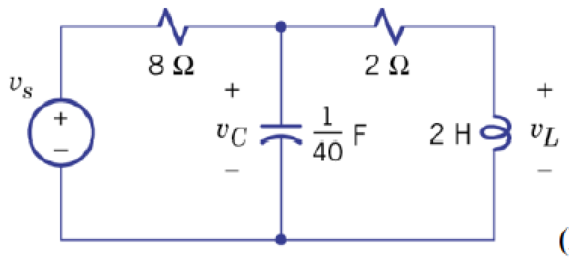
(b) (10 pts) Calculate V_C(s), Laplace transform of $v_c(t)$, and I_L(s), Laplace transform of $i_L(t)$, from the s-domain circuit diagram for t ≥ 0.

(c) (10 pts) Apply partial fraction expansion and inverse Laplace transform to determine $v_C(t)$ and $i_L(t)$ for t ≥ 0. <u>If the response contains certain sinusoidal function, be sure to express it as the standard-form cosine function $A\cos(Bt+\phi)$</u>.



Solution

$$v_s(t) = \begin{cases} 20V, & t < 0 \\ 8e^{-t}V, & t > 0 \end{cases}$$



(Fig. 2)

(a) When $t > 0$

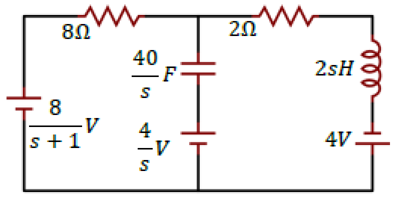
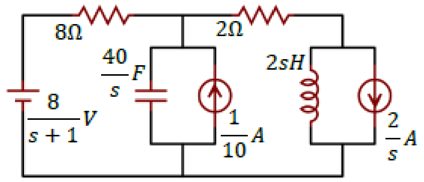
$$\textbf{(2\%)}\ \boldsymbol{v_s(t) = 8e^{-t}V \rightarrow V_s(s) = \frac{8}{s+1}}$$

$$\textbf{(2\%)}\ \boldsymbol{v_C(0^-) = 4V}$$

$$\textbf{(2\%)}\ \boldsymbol{s - domain\ of\ C: \begin{cases} \frac{40}{s}F + \frac{4}{s}V \\ \frac{40}{s}F // \frac{1}{10}A \end{cases}}$$

$$\textbf{(2\%)}\ \boldsymbol{i_L(0^-) = 2A}$$

$$\textbf{(2\%)}\ \boldsymbol{s - domain\ of\ L: \begin{cases} 2sH + 4V \\ 2sH // \frac{2}{s}A \end{cases}}$$

<table>
<tr><th>s-domain Thevenin Model</th><th>s-domain Norton Model</th></tr>
<tr><td>
</td><td>
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(4%)

(b)

$$\textbf{(4\%)}\ \frac{V_c(s)-\frac{8}{s+1}}{8}+\frac{V_c(s)-\frac{4}{s}}{\frac{40}{s}}+\frac{V_c(s)+4}{2s+2}=0 \text{ (By KCL)}$$

$$\textbf{(3\%)}\ V_c(s)=\frac{4s-36}{s^2+6s+25}$$

$$\textbf{(3\%)}\ I_L(s)=\frac{\frac{4s-36}{s^2+6s+25}+4}{2s+2}=\frac{2s^2+14s+32}{(s+1)(s^2+6s+25)}$$

(c)

$$\textbf{(2\%)}\ V_c(s)=\frac{4s-36}{s^2+6s+25}=\frac{4(s+3)-12\cdot 4}{(s+3)^2+4^2}$$

$$\textbf{(3\%)}\ v_C(t)=4e^{-3t}\cos 4t-12e^{-3t}\sin 4t$$

$$=4\sqrt{10}e^{-3t}\cos(4t+71.57°)$$

$$\textbf{(2\%)}\ I_L(s)=\frac{1}{s+1}+\frac{s+7}{s^2+6s+25}=\frac{1}{s+1}+\frac{(s+3)+4}{(s+3)^2+4^2}$$

$$\textbf{(3\%)}\ i_L(t)=e^{-t}+e^{-3t}\cos 4t-e^{-3t}\sin 4t$$

$$=e^{-t}+\sqrt{2}e^{-3t}\cos(4t-45°)$$

TABLE 13.1 Laplace Transform Properties

Operation	Time Function	Laplace Transform
Linear combination	$Af(t) + Bg(t)$	$AF(s) + BG(s)$
Multiplication by e^{-at}	$e^{-at}f(t)$	$F(s + a)$
Multiplication by t	$tf(t)$	$-dF(s)/ds$
Time delay	$f(t - t_0)u(t - t_0)$	$e^{-st_0}F(s)$
Differentiation	$f'(t)$	$sF(s) - f(0^-)$
	$f''(t)$	$s^2F(s) - sf(0^-) - f'(0^-)$
Integration	$\int_{0^-}^{t} f(\lambda)\, d\lambda$	$\frac{1}{s} F(s)$

TABLE 13.2 Laplace Transform Pairs

$f(t)$	$F(s)$
A	$\frac{A}{s}$
$u(t) - u(t - D)$	$\frac{1 - e^{-sD}}{s}$
t	$\frac{1}{s^2}$
t^r	$\frac{r!}{s^{r+1}}$
e^{-at}	$\frac{1}{s + a}$
te^{-at}	$\frac{1}{(s + a)^2}$
$t^r e^{-at}$	$\frac{r!}{(s + a)^{r+1}}$
$\sin \beta t$	$\frac{\beta}{s^2 + \beta^2}$
$\cos (\beta t + \phi)$	$\frac{s \cos \phi - \beta \sin \phi}{s^2 + \beta^2}$
$e^{-at} \cos (\beta t + \phi)$	$\frac{(s + a) \cos \phi - \beta \sin \phi}{(s + a)^2 + \beta^2}$